ASOP-3: A Program for Optimum Structural Design to Satisfy Strength and Deflection Constraints

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Optimality criteria are applied in the iterative resizing of the elements of a finite-element model to achieve minimum weight. In addition to materials that can be considered to be homogeneous for analysis purposes, composite laminates with layups of considerable generality can be treated. Strength resizing of composite elements is done by treating the laminate as a unit, permitting the application of criteria consistent with current design practice. In addition, the program nominally treats a single generalized deflection constraint (linear combination of nodal deflections), but multiple constraints can be accommodated in many cases of practical interest by multiple submissions of the program. Results are presented for two representative lifting-surface structures, subject to both strength and twist constraints.

Introduction

FULL realization of the potential of advanced composite materials for efficient aerospace structural design from a strength standpoint and the achievement of beneficial passive deformation in lifting surfaces requires the availability of powerful and convenient automated design tools. In addition to other contributions in this area,[1-3] recent efforts in the development of the series of computer programs with the designation ASOP (Automated Structural Optimization Program), which initially emphasized metallic structures,[4] have been directed toward the provision of such tools for the design of composite structures.[5,6]

The present paper describes the capabilities of ASOP-3,¶ the most recent program in that series. The program operates on a finite-element model of a structure that can be highly detailed (up to 1000 nodes and 3000 finite elements). Its principal features are its capability of accommodating laminates of rather general layup—treating the laminate as a unit in strength resizing and applying practical design criteria—and its capability of resizing both noncomposite and composite structures for a generalized deflection constraint representing a linear combination of translational displacements of structural nodes. The generalized deflection constraint might consist, for example, of lifting surface twist or camber at a specified station. When laminates are used, it is possible to balance the numbers of laminae in specified pairs of fiber directions.

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¶The ASOP-3 program is presently operational on the CDC CYBER 74 and CYBER 172 computing systems. Its distribution is restricted. Inquiries concerning its availability should be addressed to the Structural Mechanics Division, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio 45433.

Program Structure

Various portions of the program are distinguished from one another on a functional basis. Prior to any resizing step, either for stress constraints or for a deflection constraint, it is necessary to analyze the structure; that is, to solve the equations relating nodal displacements to applied loads, and to determine the corresponding internal loads in individual elements. Accordingly, one portion of the program is concerned with such analysis. It should be noted that the applied loads must be supplied by the user. The program does not have the capability of determining the applied loads corrected for aeroelastic deformations. Another portion of the program takes the internal loads computed in this analysis and uses them to resize the elements for stress constraints and constraints imposed by specified minimum and/or maximum gages. It should be noted that minimum gages usually represent limitations associated with practical construction, while the maximum gage is normally used as a means of fixing an element's gage by setting its minimum and maximum gages equal to one another. A third portion of the program uses the nodal deflections computed in the analysis part of the program in resizing the elements for an imposed deflection constraint, taking cognizance also of the stress constraints and specified minimum and/or maximum gages.

When a deflection constraint is to be applied, two different phases, or "modes," in the redesign process are distinguished. In the "stress-constraint mode," which is the one that is executed first, a number of cycles of analysis and resizing for stress and minimum or maximum gage constraints are performed, until a convergence criterion is satisfied or the number of cycles has reached a specified maximum. The design should then be fully stressed or nearly fully stressed.

The "deflection-constraint mode" is then entered with that design, and deflection-constraint resizing and stress-constraint resizing are done sequentially within each cycle in that mode, with an analysis following each type of resizing. There are therefore two analyses performed in each cycle in the deflection-constraint mode. This cycling in the deflection-constraint mode is continued until a convergence criterion is satisfied or the number of cycles has reached a specified maximum.

Structural Analysis and Modeling

The analysis portion of the program, in which nodal displacements and internal loads (element nodal forces) are determined for given external loading, assumes linear elastic behavior of the structure and applies the matrix displacement method of finite-element structural analysis.[7] Up to twenty applied loading conditions can be treated.

The types of finite elements that can be accommodated include uniform-strain bars and triangular membrane elements, beam elements, quadrilateral membrane elements comprising an assemblage of four uniform-strain triangles, and quadrilateral shear elements based on Garvey's assumptions.[8] A moderate amount of warpage is permissible in the quadrilateral membrane and shear elements.

In the modeling of structures constructed of filamentary composite materials, membrane elements are used exclusively in the program to represent laminates with a variety of fiber directions. Composite elements can have any number of layers up to a maximum of six, where a "layer" is defined as the aggregate of all laminae of a given composite material with fibers in a given direction. The fiber directions for the different layers can be arbitrary, except that, in aggregate, the layers should constitute a laminate whose strength is fiber-controlled. A fiber-controlled laminate is one in which the fiber directions are sufficient in number and distribution that any internal loading can be carried by fibers alone.

Layers of composite elements are treated internally in the program as separate elements, except in resizing for stress constraints, where the whole laminate is treated as a unit, as discussed later. The layers of a composite element thus constitute a stack of elements that are connected only at the corner nodes.

The stiffness matrix of each individual element (or layer of a composite element) is determined at the outset for a unit value of the corresponding design variable (cross-sectional area of a bar or beam element and thickness of a membrane or shear element). The stiffness matrices of all the elements are then multiplied by the corresponding design variables and "stacked" to form the stiffness matrix of the whole structure.

The assumption that the stiffness matrix of each element is proportional to its design variable is clearly valid in the case of bar, membrane, and shear elements. Its validity in the case of beam elements is based on the assumption that the radii of gyration of the cross section of such an element remain constant with changes in cross-sectional area.

After formation of the structure stiffness matrix $[K]$, the matrix equation

$$[K][\Delta] = [P] \tag{1}$$

in which $[P]$ is the matrix of applied loads, is solved for the corresponding nodal displacements $[\Delta]$. The displacements obtained in this manner are then used in conjunction with the element stiffness matrices to determine the internal nodal forces acting on the individual elements. These nodal forces are then used in the determination of average stresses in the elements.

In the case of composite elements, the stiffness matrix for each layer is determined on the basis of the full stiffness properties of the material, including contributions from both fiber and matrix material. The internal loads are then based on these properties. This is emphasized, because the stiffness of the matrix material is neglected in subsequent calculations associated with the resizing process.

The Stress-Constraint Mode

In the stress-constraint mode, an initial design, expressed as a set of specified values of the design variables, is selected, and an analysis to determine nodal deflections and element stresses is carried out for a given set of applied loading conditions. The states of stress in each element for the various loading conditions are then used to determine whether the element is understressed or overstressed, and the design variable for that element is then adjusted accordingly. This process of analysis and resizing is repeated cyclically until a given number of cycles have been performed or until convergence has been achieved. A converged design is referred to as a fully stressed design, that is, one in which each element is stressed to the maximum allowable extent in at least one loading condition without being overstressed in any loading condition, or is at a minimum or maximum prescribed gage.

In the case of noncomposite elements, the state of stress in each element for each loading condition is used in conjunction with a failure criterion to determine a "stress ratio," which provides a measure of the extent to which the stress constraint is satisfied or violated. It is equal to unity if the failure criterion is exactly satisfied. The maximum value of the stress ratio for all loading conditions for each element is then used as a multiplying factor in resizing the design variable for that element.

The criteria governing the failure of composites are more complex than those governing the failure of noncomposite materials. Consequently, the algorithm for composite element resizing is necessarily more complex, requiring that the laminate be treated as a unit, so that interaction between layers may be properly taken into account. Furthermore, because operational experience with composite materials is still quite limited, it is desirable to make certain conservative assumptions concerning their strength behavior.

For example, local cracking or crazing in the matrix material may greatly reduce its effectiveness as a load-carrying agent, even though it continues to serve its central purpose as a binding agent. For that reason, the assumption is made in resizing the layers of a composite element that all the load is carried by the fibers, an assumption that is valid only for fiber-controlled composites.

Another conservatism, one that may be applied as an option in the program, relates to interaction between laminate layers that is somewhat akin to the effect of hydrostatic stress in metals or other nominally homogeneous materials. It is known, for example, that if components N_x and N_y of the laminate stress resultant are both present and of the same sign, some layers will be less severely stressed than if either N_x or N_y were absent. Some designers feel that it is unconservative to take advantage of this fact and base the design on the simultaneous presence of both components. The dynamic nature of some loading conditions suggests the possibility that different components of the internal loading may not be applied simultaneously. The program option, referred to as the "cutoff" option, makes it possible to provide additional stress checks with N_x and N_y set successively to zero, and to use the largest resulting fiber stresses in the resizing process.

In addition to fiber failure in tension or compression, the possibility of failure in the so-called "microbuckling" mode may be taken into account. In that mode, there is a highly localized buckling of the fibers because of their own low bending stiffness and the limited shear stiffness of the matrix material, which is relied upon to resist such buckling.[9]

In the absence of a direct method for redesigning a laminate layup based on the strength criteria just discussed, it is necessary to use an iterative procedure. The element stress resultants obtained from the analysis of the whole structure in the current cycle and the laminate layup used in that analysis represent the starting information. Fiber stresses are then determined on the basis of a laminate analysis in which the stiffness of the matrix material is neglected, thus discounting the load-carrying capability of the matrix material. It will be recalled, however, that in the analysis of the whole structure to determine nodal displacements and internal loads, the stiffness of the matrix material is not neglected. The fiber stresses are then used in conjunction with the allowable fiber stress to resize the laminate layers, rounding each layer thickness up to the nearest multiple of a lamina thickness. As the fiber stresses were based on the old layup, it is necessary to repeat this procedure, using the same laminate stress resultants but the new layup. This is done iteratively until convergence is achieved. The resulting layup represents an optimum design for the given internal loading. The next analysis of the whole structure then provides a new distribution of the internal loads.

A further advantage of treating the laminate as a unit in resizing is that it provides a suitable framework for the introduction of failure criteria for compressive instability other than microbuckling. While the program does not, at present, have the capability of treating such additional buckling modes for composites, the existence of this framework would facilitate the introduction of such a capability.

The Deflection-Constraint Mode

Resizing for deflection constraints is accomplished in ASOP-3 by the application of an optimality criterion. It has been shown[10,11] that, for the case of a single deflection constraint, and in the absence of other constraints, a minimum-weight design is achieved when the partial derivative of the constrained deflection with respect to element weight has the same value for all elements. That is,

$$\partial\delta/\partial w_i = C \qquad (i=1,2,....n) \tag{2}$$

where δ is the deflection to be constrained, w_i is the weight of the ith element (of a total of n elements), and C is a constant.

When, as in most practical designs, there are strength constraints and minimum and/or maximum gage constraints in addition to the deflection constraint, the uniform-derivative criterion expressed in Eq. (2) can be applied to the set of all elements not governed by these other constraints, the corresponding element weights being referred to as the "active" variables. In that case, the criterion is less rigorously applicable, but should still give a design of nearly minimum weight.

The minimum-weight design cannot be arrived at directly. It is necessary to employ an iterative process which has been found to converge rapidly in practical cases. The iterative process used in ASOP-3 is similar, but not identical, to that used in ASOP-2[5] and in a number of other references, including Refs. 12, 13, and 14. In the details of its application, it resembles most closely the procedure used in the FASTOP program for optimization to satisfy a constraint on flutter velocity.[11]

Starting with a nonoptimum design, which may or may not satisfy the prescribed deflection constraint, the following recursion relation is applied in successive cycles:

$$w_{i_{\text{new}}} = w_{i_{\text{old}}}\sqrt{\frac{(\partial\delta/\partial w_i)_{\text{old}}}{(\partial\delta/\partial w)_{\text{target}}}} \tag{3}$$

where $w_{i_{\text{old}}}$ is the weight of the ith element prior to resizing in the current cycle, $w_{i_{\text{new}}}$ is the weight of the ith element following resizing in the current cycle, $(\partial\delta/\partial w_i)_{\text{old}}$ is the partial derivative of the constrained deflection with respect to w_i, computed for the design existing prior to resizing in the current cycle, and $(\partial\delta/\partial w)_{\text{target}}$ is a quantity called the "target derivative" and is defined below. The determination of the derivatives $\partial\delta/\partial w_i$ is discussed in the Appendix.

At the optimum design, the target derivative $(\partial\delta/\partial w)_{\text{target}}$ will be the constant C in Eq. (2), and the derivatives $\partial\delta/\partial w_i$ will all be equal to it. However, prior to convergence to an optimum design, the derivatives, $\partial\delta/\partial w_i$, will differ from each other in value and, in fact, may differ in sign. Depending upon the sign of the target derivative, some of these derivatives may then yield a negative value for the quantity under the radical in Eq. (3). The corresponding elements will then have to be excluded when Eq. (3) is applied, and their gages will be set at the values determined in the latest resizing for strength, subject also to specified minimum and/or maximum gages. Element gages determined in the latest resizing for strength are also applied as minima to those elements that are resized by Eq. (3), and the gages yielded by that equation are subject to revision on the basis of minimum and/or maximum permissible values.

As the value of C in Eq. (2) is not known until the optimum design is achieved, it is necessary to find a value for the target derivative that, when introduced into Eq. (3), will yield a design that satisfies the constraint, at least approximately. This is done by a trial procedure in which a value of the target derivative is sought that will yield a design satisfying the relation

$$\delta_{\text{desired}} = \delta_{\text{old}} + \sum_{i=1}^{n} \frac{1}{2}\left[\left(\frac{\partial\delta}{\partial w_i}\right)_{\text{old}} + \left(\frac{\partial\delta}{\partial w}\right)_{\text{target}}\right] \times (w_{i_{\text{new}}} - w_{i_{\text{old}}}) \tag{4}$$

where δ_{old} is the value of the constrained deflection prior to resizing, δ_{desired} is the desired value of the constrained deflection in a resizing step, and the summation is over all elements in the model, including those governed by constraints other than the deflection constraint.

Equation (4) provides a second-order approximation (in the Taylor series sense) to the desired value of the deflection. An exact value could have been obtained by taking the new design and performing a structural analysis to determine the deflection subject to constraint. However, as this operation would have to be performed a number of times for successive trial values of the target derivative and is expensive computationally, it is highly advantageous and, in practice, satisfactory to use Eq. (4) instead.

It was stated previously that Eq. (3) is applied to that group of elements with derivatives, $\partial\delta/\partial w_i$, that are of the same sign as the target derivative. The determination of that sign is now considered. It is established upon entry into the deflection-constraint mode and depends upon whether the constraint value of the subject deflection exceeds or is less than its current value (for the design existing upon entry into the deflection-constraint mode). If the constraint value exceeds the current value algebraically and the constraint is either (1) an equality constraint or (2) an inequality constraint that has been violated, an increase in deflection is then desired, and the proper derivative sign is that which is associated with an increase in deflection resulting from an increase in element weight, that is, a positive sign. For those elements with negative derivatives, a reduction in element weight will move the deflection in the desired direction, and the design variables for these elements can be permitted to decrease to the extent permitted by other constraints. In the reverse situation—where the constraint value is less than the current value—Eq. (3) is applied to those elements with negative derivatives. Where the constraint is an inequality constraint and is not violated by the design existing at exit from the stress-constraint mode, no further resizing is necessary.

Once the determination of the sign of the derivatives to be introduced into Eq. (3) is made, it remains unchanged throughout the remainder of the procedure. As long as movement from the initial value of the constrained deflection (upon entry into the deflection-constraint mode) toward the constraint value is in the same direction, it is clear that this sign should not be changed. However, what about the situation where the constraint value is overshot and movement in the revrrse direction is necessary? If the sign were to be reversed, all those elements previously resized by the deflection constraint would be suddenly relieved of such constraint and their gages could drop to values determined by other constraints. Under these circumstances, large changes could be expected to result from a need for minor adjustments, as the amount of overshoot would normally be small. These large changes could be expected to preclude satisfactory convergence to an optimum design. Maintaining the same sign keeps these adjustments essentially within the same group of elements that have previously been governed by the deflection constraint.

The sign of the target derivative will, by definition, be the same as that of the derivatives $(\partial\delta/\partial w_i)_{old}$ introduced in Eq. (3). It remains to find a value of the target derivative that will satisfy Eq. (4). This is done by arbitrarily taking as an initial trial value, upon entry into the deflection-constraint mode, a value equal to 80% of the average of all the derivatives having the proper sign, determined as previously explained. (In subsequent redesign cycles of the iterative redesign process, as will be explained, the starting value of the target derivative is the last value computed in the preceding cycle.) The target derivative is then incremented until a value is achieved that satisfies Eq. (4) within a tolerance specified by the user.

The value of $\delta_{desired}$ in Eq. (4) is not necessarily the constraint value. It may be advantageous in some situations to move from the initial value of the subject deflection to the vicinity of the constraint value in a series of shorter steps, rather than in a single step. Equation (4) then provides a closer approximation in each step. Furthermore, when the near vicinity of the constraint boundary in the design space is reached, it may be at a point considerably closer to the minimum-weight design point. Accordingly, the program provides an option that permits the change from the initial value of the deflection to the constraint value to be made in a number of approximately equal increments, that number being selected by the user.

The deflection that is subject to constraint may be generalized, in the sense that it may be represented as a linear combination of nodal displacements in specified degrees of freedom. Thus, for example, an angular displacement constraint may be treated by representing it as the difference between the translational displacements of two specified points divided by the distance between them. The two points specified need not be at nodes; their displacements can be obtained by interpolation between nodal displacements. Similarly, a given amount of camber of a lifting surface at a given spanwise station can be specified as a constraint by a similar representation as a linear combination of nodal displacements.

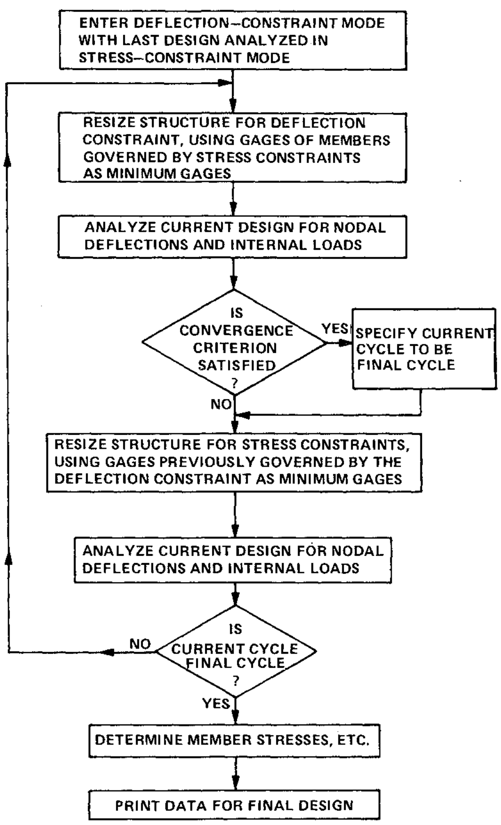

Fig. 1 Iteration cycles in deflection-constraint mode.

When composite elements are included in the model, each layer of such elements is treated internally in the program as a separate element. Accordingly, Eq. (3) is applied to individual layers, with the deflection derivative being computed for each layer with respect to that layer's weight. In the trial process of finding a value of the target derivative that satisfies Eq. (4), the layer thickness is not rounded. Rounding is done only after that process is completed, when the layer thickness is rounded up or down to the nearest multiple of the lamina thickness. By rounding both up and down, the effect of rounding on the constrained deflection is minimized.

In the deflection-constraint mode, deflection-constraint and stress-constraint resizing are performed in alternating fashion, as illustrated in the flow chart shown in Fig. 1. The interaction between the two types of constraint is taken into account by treating element gages that are governed by stress constraints in a stress-constraint resizing step as minimum gages in the next deflection-constraint resizing step, and similarly treating element gages that are governed by the deflection constraint in a deflection-constraint resizing step as minimum gages in the next stress-constraint resizing step. This process converges to a design in which there are two classes of elements (or layers in the case of composites). One class comprises elements that are fully stressed or are at minimum or maximum specified gage. The other class comprises elements that are governed by the deflection constraint. In this latter class, the derivatives of deflection with respect to element (or layer) weight all have nearly uniform values. Departures from uniformity are due to lack of convergence or, in the case of composite element layers, to rounding to an integral number of laminae. Under these circumstances, it can be expected that the design will be close to optimum, at least in a local, if not in a global, sense.

While deflection-constraint resizing is presently limited to a single constraint in a single loading condition (although as many as twenty loading conditions can be applied in stress-constraint resizing), this limitation can be effectively circumvented in special cases. In particular, the case of a cantilever structure, such as a high-aspect-ratio wing or tail surface, with constraints on angle of twist at several spanwise stations, can be treated. This is done by means of successive submissions of the program. In the first submission, only the innermost constraint is applied. A second submission is then made, in which only the next outboard constraint is applied, with the gages of all members inboard of the innermost constraint location being kept fixed in deflection-constraint resizing at the values yielded by the first submission. In subsequent submissions, constraints are applied at successive locations, moving outboard, each time keeping all gages of members inboard of the last constrained station fixed at their latest values. This procedure cannot be expected to yield an exact result, but it should provide a design that satisfies the constraints approximately.

Applications to Representative Problems

The ASOP-3 program has been applied to two lifting-surface structures to demonstrate its capability for both stress-constraint resizing and deflection-constraint resizing. The two models representing these structures are of quite different levels of detail—one being relatively coarse and serving primarily to demonstrate the resizing capability of the program as it would be applied at the preliminary design stage, the other being a much more refined model demonstrating the program's capability of handling large problems, such as would be encountered at a more detailed design stage. The two models and the results obtained for them are now described.

Preliminary Wing Design

The cantilevered wing model, shown in planform in Fig. 2, was chosen for study as an illustration of the application of

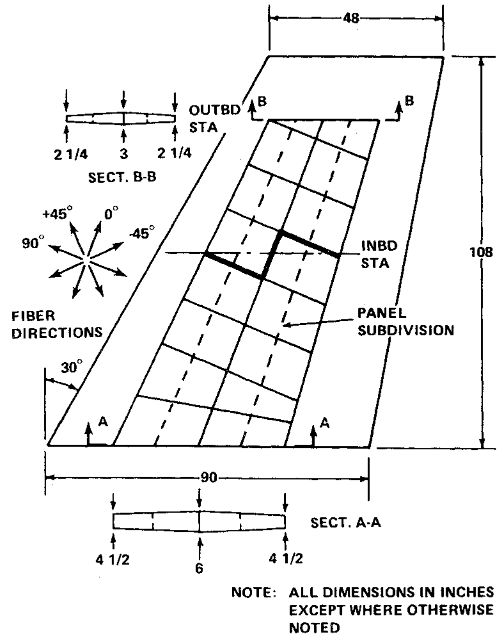

Fig. 2 Aerodynamic planform and primary structural arrangement of preliminary wing design.

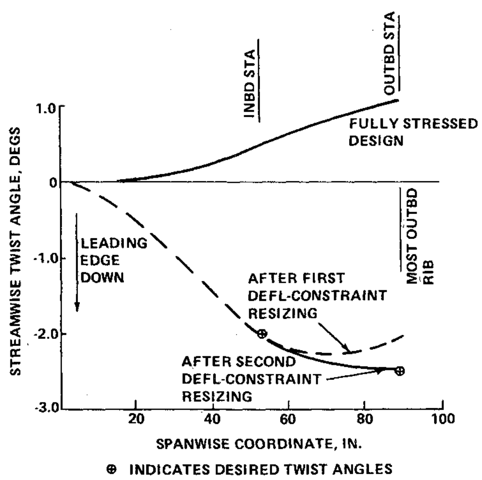

Fig. 3 Preliminary wing design—streamwise twist distribution at various stages of resizing.

the program in the preliminary design of a lifting surface. The primary structure is a symmetric two-cell box beam having aluminum substructure and graphite/epoxy cover skins with fiber directions as shown in Fig. 2. The solid lines indicate the locations of shear webs. The substructure is modeled with conventional shear-panel elements and posts (which are the only bar elements in the model); the cover skins are composite four-layer elements which are permitted to have unbalanced ±45deg layers. The model has 88 nodes and 158 members and is fixed at the root.

Two applied loading conditions were generated by using simplified pressure distributions representative of a subsonic, forward-center-of-pressure loading and a supersonic, near-uniform-pressure loading. The structure was sized for these conditions in the stress-constraint mode without exercising the "cutoff" option described in the section titled "The Stress-Constraint Mode." Satisfactory convergence was achieved in five cycles in the stress-constraint mode. The resulting design was then examined from the point-of-view of streamwise-twist distribution along the wing's span for the subsonic condition—the twist angle being based simply on the difference in vertical displacements between the forward and aft

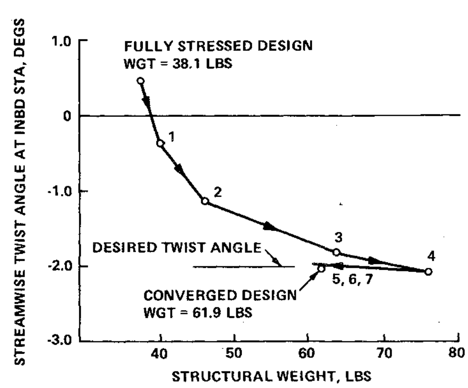

Fig. 4 Preliminary wing design—resizing history from fully stressed design to deflection-constrained design for inboard station.

wing spars along a streamwise chord. This twist distribution is shown by the upper curve in Fig. 3. It is interesting to note that the forward c.p. of the subsonic loading distribution causes sufficient nose-up twisting to overpower the usual nose-down twisting (washout) that generally occurs in swept metallic wings.

To illustrate a potential application of the deflection-constraint resizing capability of ASOP-3, it was decided to attempt to "tailor" the design to achieve a prescribed streamwise-twist distribution for the subsonic loading condition that would offer improved aerodynamic performance through increased lift-to-drag ratio. Twist angles at two wing stations were then established as targets, these being −2.0 deg (washout) at a selected inboard station and −2.5 deg at the most outboard rib station (see Figs. 2 and 3).

Resizing in the deflection-constraint mode was accomplished in two stages. The approach was to divide the structure into two regions, as indicated by the bold separating line in Fig. 2. In the first resizing stage, only the composite cover-skin elements in the inboard region were permitted to be resized in the deflection-constraint part of a resizing cycle, to meet the inboard station twist-angle requirement. In the second stage, only outboard region cover-skin elements were allowed to be resized, to achieve the desired outboard-station twist angle. In both stages, however, all elements were permitted to be resized if they were strength critical. This two-stage approach was based on the concept that, for high-aspect-ratio cantilevered surfaces, the resizing of elements outboard of a particular station should have little influence on the deflections at that station.

The first stage of resizing in the deflection-constraint mode started with the fully stressed design. Convergence to the desired twist angle at the inboard station was achieved in seven steps with the overall resulting twist distribution as shown by the dashed curve in Fig. 3. Figure 4 summarizes the resizing history in this mode, in terms of inboard station twist angle versus total structural weight, after the strength resizing part of each cycle.

In the second stage of deflection-constraint resizing, all starting gages were taken as those of the final design in the previous run. For elements in the inboard region, these starting gages were also treated as minimums to prevent removal of material that was previously introduced to meet the inboard station twist requirement. Convergence to the desired outboard station twist angle required only two cycles, with only a very small additional weight increase. The final twist distribution after this second deflection-constraint resizing is shown in Fig. 3, and a summary of results for all stages of resizing is presented in Table 1. It should be pointed out that the small differences between the target and accepted twist angles are due mainly to limits imposed by the practical requirement for rounding layers to integral numbers of laminae. Figure 5 displays the cover-skin layer arrangement for the initial fully stressed design and the final combined

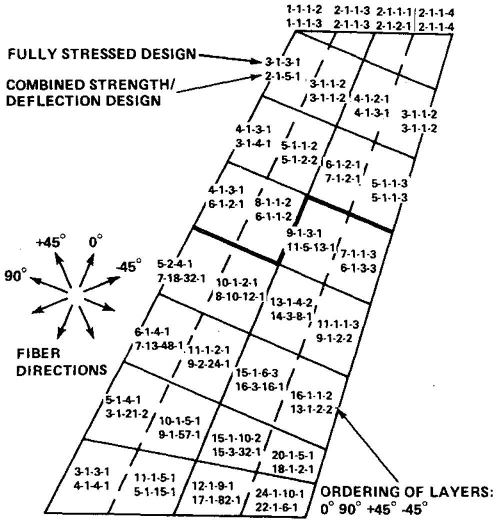

Fig. 5 Cover-skin layups for preliminary wing design.

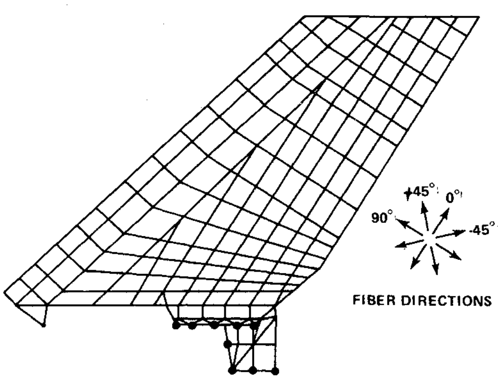

Fig. 6 Bomber fin model.

strength and deflection-constrained design. It should be noted that, because of geometric symmetry and the use of equal allowable stresses in tension and compression, the design is symmetric about the middle surface.

Bomber Fin

The second model studied is that of a bomber fin, shown in planform in Fig. 6. This model was supplied by the Air Force Flight Dynamics Laboratory and is a derivative of one that was used in the early design stages of an actual structure.

The support points of the model are shown as heavy dots in Fig. 6. The forward support point is a single point on the structure's plane of symmetry. The remaining support points occur as pairs, symmetrically located with respect to the plane of symmetry. Shear webs and cover bar elements are used to simulate spars along all of the spanwise grid lines and to simulate ribs at the root, tip, and several intermediate locations. Posts are present at all of the grid points. All of these elements are fixed in gage and are of isotropic material with the Young's modulus of aluminum. The same is true of the cover membrane elements in the two spanwise bays closest to the leading edge, as well as in the extended root structure. The remaining cover elements are of graphite/epoxy composite with fiber directions as shown in Fig. 6, the +45 deg and −45 deg layers being allowed to be unbalanced. These composite elements are the only ones that are adjustable in the design. The model contains 375 nodes and 1293 members.

Five cycles of resizing were performed in the stress-constraint mode and an additional five cycles in the deflection-constraint mode. The stress-constraint resizing was based on eight applied-loading conditions corresponding to a wide

Table 1 Summary of results for preliminary wing design

Constraint mode	Cycles to convergence	$\theta_{\text{Inbd, deg}}{}^{a}$	$\theta_{\text{Outbd, deg}}{}^{b}$	Structural weight, lb
Stress	5	+0.47	+1.06	38.1
First deflection	7	−2.03	−2.05	61.9
Second deflection	2	−2.04	−2.43	62.3

[a] Desired value = −2.00 deg. [b] Desired value = −2.50 deg.

Table 2 Summary of results for bomber fin

Constraint mode	Cycles	Tip incidence, deg	Structural weight, lb
Stress	5	−1.07	679.3
Deflection	5	0.04	691.1

variety of flight conditions. The load distribution in each of these conditions was made precisely antisymmetric about the plane of symmetry of the fin to insure that a symmetric design would be obtained, as the program does not presently provide an explicit means of enforcing symmetry. It should be noted that an alternate way of obtaining a symmetric design, without a restriction to antisymmetric loading, would be to introduce the loading conditions as matched pairs in which each condition is the mirror image of the other.

The deflection constraint applied was on the angular displacement of the tip chord relative to the root. It was applied in a loading condition in which the strength-governed design experiences a washout of 1.07 deg at the tip. The constraint was applied to eliminate this washout, so that the fin would be fully effective in this design condition; that is, it would have the same aerodynamic characteristics as a rigid structure.

It is seen from Table 2 that satisfaction of the deflection constraint required a total weight increase of only 1.7%. The reason for the smallness of this increase, as a percentage of the total weight, is twofold. First, the increase in composite thickness in the three spanwise bays closest to the trailing edge is partially offset by decreases in thickness in the more forward bays. Second, the composite cover material represents a relatively small part of the total weight of this structure. The increase in weight, as a percentage of the composite cover weight at the end of the stress-constraint mode, was 27.6%.

It should be noted that, while five cycles were performed in the deflection-constraint mode, good convergence was obtained after only four cycles, with the vicinity of the constraint value of the deflection being approached in a single step.

Concluding Remarks

The ASOP-3 program puts together in one package a comprehensive, practical capability for treating complex structures, including structures constructed of filamentary composites of rather general layup, subjected to both strength and deflection constraints. Its use of optimality criteria has proven very effective in the treatment of large numbers of design variables. It resizes composites for strength by treating the laminate as a unit, making possible the application of criteria based on current design practice and providing a suitable framework for future introduction of failure criteria for compressive instability modes other than microbuckling. Furthermore, in resizing composites for both strength and deflection constraints, balancing of specified layers can be maintained.

Appendix

The relations needed in the determination of the partial derivatives of the generalized deflection δ subject to constraint, with respect to the element weights w_i, are now derived. Starting with the basic equation relating the applied loads $\{P\}$ to the associated nodal displacements $\{\delta^p\}$,

$$\{P\} = [K]\{\delta^p\} \tag{A1}$$

the partial derivatives of the applied loads with respect to the weight w_i of the ith element, are formed, as follows:

$$\frac{\partial\{P\}}{\partial w_i} = \frac{\partial[K]}{\partial w_i}\{\delta^p\} + [K]\frac{\partial\{\delta^p\}}{\partial w_i} \tag{A2}$$

Both sides of Eq. (A2) are identically zero, because the applied loads are not functions of the design variables; thus,

$$[K]\frac{\partial\{\delta^p\}}{\partial w_i} = -\frac{\partial[K]}{\partial w_i}\{\delta^p\} \tag{A3}$$

The generalized deflection is now written in the form

$$\delta = \{\bar{Q}\}^{\mathrm{T}}\{\delta^p\} \tag{A4}$$

where $\{\bar{Q}\}$ is a vector in which the weighting coefficients are placed in the locations corresponding to the degrees of freedom of the deflections they multiply.

The partial derivative of δ with respect to w_i is given by

$$\frac{\partial\delta}{\partial w_i} = \{\bar{Q}\}^{\mathrm{T}}\frac{\partial\{\delta^p\}}{\partial w_i} \tag{A5}$$

If $\{\bar{Q}\}$ is regarded as a "virtual load" vector, a corresponding "virtual displacement" vector $\{\delta^q\}$ may be determined by solution of the equation

$$[K]\{\delta^q\} = \{\bar{Q}\} \tag{A6}$$

Premultiplication of both sides of Eq. (A3) by $\{\delta^q\}^{\mathrm{T}}$ yields

$$\{\delta^q\}^{\mathrm{T}}[K]\frac{\partial\{\delta^p\}}{\partial w_i} = -\{\delta^q\}^{\mathrm{T}}\frac{\partial[K]}{\partial w_i}\{\delta^p\} \tag{A7}$$

From Eqs. (A5) and (A6), it can be seen that the left-hand side of Eq. (A7) is simply $\partial\delta/\partial w_i$. Furthermore, if the elements of the stiffness matrix $[K]$ are linear functions of the design variables w_i, as they usually are, and $[k_i]$ is the stiffness matrix of the ith element for unit value of w_i, $\partial[K]/\partial w_i$ is seen to be equal to $[k_i]$. Thus, Eq. (A7) may be written in the form

$$\partial\delta/\partial w_i = -\{\delta^q\}^{\mathrm{T}}[k_i]\{\delta^p\} \tag{A8}$$

where $\{\delta^q\}$ and $\{\delta^p\}$ are compressed to contain only the degrees of freedom associated with member i.

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